MACIEJ K. DUDEK*

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Equilibrium Discrepancy between GDP and GNP in Poland

Introduction

In this paper we undertake the task of assessing the level of economic activity of Polish citizens in the long run. Specifically, we acknowledge the presence of a discrepancy between the GDP and GNP in Poland given the sizable inflows of foreign capital to Poland and relatively limited outflows of Polish capital to other countries. Accordingly, we identify the GNP and not the more widely publicized GDP as a measure that more accurately describes the level of economic activity in any country, and in Poland in particular. We believe that GNP rather than GDP can serve as a better predictor of the actual welfare in a given country and accordingly attempt to assess the difference between the GDP and GNP in Poland.

It is customary to note that the overall performance of the Polish economy has been successful. However, the assessment changes when the performance is viewed in relative rather than absolute terms. A simple inspection of official data allows to establish that Poland has outperformed only two out of its seven neighbors in the last fifteen years. Moreover, in relative sense Poland has lost to all its neighbors with the exception of Germany, which has been all along a mature economy. Naturally, Poland has been able to partially eliminate the distance to Germany, but the speed of convergence has been lower than expected. Furthermore, the Czech Republic, which was "richer" than Poland in 2000 continues to be "richer" today, i.e., Poland has failed to make up ground to the Czech Republic. The situation is even more dramatic in the case of Lithuania and Slovakia. Both of those economies started at levels of output per capita below that of Poland. Today the two countries are characterized by higher per capita levels of income, i.e., Poland, in this case, has failed to preserve its relative income advantage and now is lagging behind the two economies. Furthermore, Belarus deserves

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* Dr hab. Maciej Dudek – Vistula University & Institute for Market, Business Cycle, and Consumption in Warsaw; e-mail: mkdudek@alumni.caltech.edu.

more praise as the income gap that existed between Poland and Belarus in 2000 has been reduced at a rate faster than the gap between Poland and Germany. In other words, Belarus has caught up with Poland to a larger extent than Poland has with Germany.

Furthermore, economic developments in Russia should be viewed as miraculous relative to the Polish experience. In the last fifteen years Russia has completely eliminated the distance to Poland and today enjoys GDP per capita higher than that noted in Poland. The GDP in Russia has grown rapidly outpacing the rate of growth in Poland. It i naturally necessary to emphasize that the Russian experience, contrary to the Polish experience, has not been dependent on foreign capital inflows. Only compared with Ukraine Poland's growth advantage is spectacular, but this "success" has been mostly driven by Ukrainian failure and not by Polish miraculous growth performance.

In the relative sense Poland cannot be perceived as a leader or winner. Furthermore, Poland, unlike her neighbors, has allowed for sizable capital inflows, and, thus, the Polish "success" can in fact be overstated. It, thus, becomes imperative to assess the true level of economic activity in Poland – a task we undertake in this paper. We establish that the actual levels of income enjoyed by Polish nationals may be expected to be even lower as there is a significant discrepancy between the GDP and GNP in Poland. We estimate, using a theoretical approach, this expected discrepancy to be equal asymptotically to about 9.5% implying a four year differential between the level of income actually earned by Polish nationals and the level of income implied by GDP.

We base our estimation procedure on a simple general equilibrium model. We allow for the presence of foreign physical capital. We calculate the equilibrium value of foreign capital and the equilibrium composition of physical capital in Poland. Specifically, we show that the value of the domestic saving rate is critical in determining the composition of physical capital in Poland. In particular, we argue that a saving rate of 18% implies that physical capital stock in Poland will be foreign owned in 37% leading to a discrepancy between the GDP and GNP of 9.5%. Furthermore, we show that increases in the domestic saving rate can be welfare improving in the long run. In particular, we show that faster domestic capital accumulation leading to a higher equilibrium capital stock and in turn influencing negatively the rental price of capital endogenously displaces foreign physical capital. Consequently, the discrepancy between the GDP and GNP is reduced and the level of domestic income rises. Finally, we show that the level of consumption in Poland can be increased in the long run by increasing the domestic saving rate. We estimate that the gains can be as large as 11%.

We would like to note that this paper is not the first one to acknowledge the shortcomings of the GDP relative to GNP as a welfare indictor see Wyżnikiewicz (2017) and Maczyńska et al. (2014). However, our contribution is the first one that utilizes formal modeling tools and, which allows us to provide long run estimates of the impact of foreign investment on the Polish economy. Furthermore, in our judgement, our paper adds to the official data on GDP and GNP published by the

Polish Statistical Office (GUS) as we account for income, in an explicit modeling framework, taking into account the factual residency of capital owners rather than legal residency registration of businesses.

The paper is organized in four sections. In the following section we report basic facts on the levels of inflow of foreign capital to Poland and provide a rough estimate of the current discrepancy between the GNP and GDP. In the next section we present our basic theoretical model that allows us to assess the discrepancies between the GDP and GNP in the long run. The last section concludes.

1. Foreign investment in Poland

We choose to utilize formal modeling tools. Such an approach naturally ensures that our exposition is logical and consistent in economic sense, but comes at cost of significant simplification, which may invariably lead to overestimates. In other words, our modeling style allows us to arrive at sharp conclusions, which are naturally fully supported by the model generated evidence, but must be taken with caution as can be exaggerated in quantitative sense and should not be considered to be final.

Finding the difference between the GDP and GNP[1], in principle, is straight-forward to establish. In particular, if one chooses to rely on a production function approach, one can express the value of the total output in a given point in time, GDP, as

$$GDP = Y = AK^{\alpha}L^{1-\alpha}. \tag{1}$$

Assuming that labor is not as mobile as capital and that holdings of capital abroad by domestic residents are relatively small, we can express (a formal derivation is provided in the subsequent section) the level of GNP as

$$GNP_t = Y_t^{PL} = Y_t\left(\alpha\frac{K_t^{PL}}{K_t^{PL} + K_t^F} + 1 - \alpha\right), \tag{2}$$

where K^{PL} denotes the level of capital held by domestic residents and K^F denotes the level of capital held by foreigners. Obviously, we have $K^{PL} + K^F = K$. The above two equations imply that

$$\frac{GNP_t}{GDP_t} = \alpha\frac{K_t^{PL}}{K_t} + 1 - \alpha, \tag{3}$$

i.e. the discrepancy between the GDP and GNP is a simple function of the share of domestic capital in the entire capital stock in an economy. Consequently, to

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[1] In this paper, we use term GNP even though many researchers find the concept of gross national income (GNI) to be more appropriate in this context. In other words, we purely judgmentally choose to ignore net transfers and only focus on net foreign income.

find the discrepancy between the GDP and the GNP we need to estimate the share of domestic capital in Poland.

Before we proceed further let us make a simple observation that it is customary to assume that entrepreneurs aim at maximizing their net residual payment, which normally takes the form of profit income. Furthermore, in a simple model with two factors of production – capital and labor – one can rely on a simple accounting identity of the form

$$Y = \pi + rK + wL, \tag{4}$$

where Y denotes – income generated, π – the level of profit earned, r – the rental price of capital and w – the wage. Naturally, rK is the total payment to capital and wL is the payment to labor.

In an isolated economy by definition all forms of income accrue to domestic agents. The situation is different if one allows for capital flows across countries. In such a case the actual distribution of income depends on the distribution of ownership. In the simplest and naturally highly extreme case we can assume that income generated by a given foreign owned entity is split according to the following rules

$$Y^F = \pi + rK \tag{5}$$

and

$$Y^{PL} = wL. \tag{6}$$

In other words, in an open economy with unhindered capital flows foreigners who own capital receive both profit and capital rental income and domestic agents are only left with labor income. As compared to the case of an isolated economy domestic agents lose in relative sense by allowing for foreign capital inflows since they have only one source of income instead of three sources of income. Obviously, this is only a relative view, and need not, and probably does not apply to absolute comparisons.

Note that foreign investment is normally assumed to generate spillover effects and is expected to enhance productivity. Therefore, it is naturally possible that foreign investment leads to a significant increase in real wages and this increase in wages can more than compensate for losses of income incurred by the change in ownership. Consequently, a given country can gain from foreign investment even when some of the sources of income are eliminated. However, whenever this happens the actual income earned by domestic residents is not as high as the level of income implied by official statistics. Thus, our objective in this paper is to assess the magnitude of the actual income earned by domestic residents.

The level of foreign investment in Poland has been significant for a number of years now. In fact, since the beginning of the transformation the total investment in Poland has exceeded 220 billion dollars. Consequently, we can expect that foreign capital stock located in Poland is significant. A rough calculation, under the assumption that the rate of depreciation of physical capital is about 7%, reveals that the value of the total capital stock owned by foreigners in Poland stands at about 109 billion dollars. Furthermore, the value of the total capital stock in

Poland is estimated at 1800 billion dollars. Therefore, we can state that the share of foreign capital in Poland is equal to 6%. Consequently, we can expect, given equation (3), that the ratio of GNP to GDP is equal, under the standard assumption that $\alpha = \dfrac{1}{3}$ to about 98% implying a very small discrepancy between the two measures of economic activity[2]. Naturally, this is only an estimate at a given point in time given the most recent data. We can expect that the magnitude of the discrepancy will evolve over time reaching eventually an equilibrium value. Our goal is now to estimate the equilibrium discrepancy between the two measures of economic activity. We embark on this task next.

2. Estimates of GNP for Poland

Given sizable presence of foreign capital in Poland – that there is a discrepancy between GDP and GNP. In this section, we attempt to assess the difference between the two measures of economic activity. Naturally, we know that GNP in Poland is likely to be smaller than GDP given that the magnitude of foreign owned capital stock in Poland is large. We are interested in estimating the actual difference. To assess the size of the gap between GDP and GNP in Poland we rely on a theoretical approach involving aggregate production function and assume optimizing behavior of foreign investors who at all times rationally build their portfolios and invest in Poland in line with their private interest. Furthermore, as noted above we agree that foreign investment generates positive spillovers and in general leads to an increase in the overall productivity. In this section, we are interested in estimating the long run difference between GDP and GNP and, accordingly, we assume that any potential productivity gains due to the presence of foreign investment have already occurred.

Typically, in modern macroeconomics one resorts to approaches that are internally consistent in the economic sense, such as general equilibrium modeling. Here we adopt this now standard approach to assess the magnitude of GNP in Poland. We start traditionally by assuming that output at any given point in time can be expressed with a production function of the form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{7}$$

where Y_t denotes a measure of output (GNP/GDP), A_t reflects a measure of productivity (TFP), K_t stands for the amount of physical capital, L_t denotes the magnitude of employment at time t, and $\alpha \in (0, 1)$ is a constant, which typically in equilibrium reflects the share of physical capital in total factor payments. Normally, it is assumed that α is equal to about $\dfrac{1}{3}$.

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[2] The official data published by the Polish Statistical Office (GUS) suggests that the actual discrepancy is closer to 3%.

Furthermore, it is traditionally assumed that goods markets and factor markets are perfectly competitive, which, in particular, implies that the equilibrium factor payments can be expressed as

$$r_t = \alpha A_t K_t^{\alpha-1} L_t^{\alpha-1}, \tag{8}$$

and

$$w_t = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha}. \tag{9}$$

Naturally, given the assumption of perfect competition the factor payments exhaust total output and we can state the following identity:

$$Y_t = w_t L_t + r_t K_t. \tag{10}$$

In the case of imperfect competition the situation becomes slightly more complex as apart from payments to the two factors (capital and labor) there are profits that constitute a share of output as well. In such a more general framework – normally based on models of monopolistic competition – the above identity can be expressed as

$$Y_t = \pi_t + w_t L_t + r_t K_t, \tag{11}$$

where π_t denotes the level of profits. In our framework to preserve simplicity we assume that $\pi_t = 0$, i.e., that profits are always zero. This is naturally only a simplifying assumption. By making this simplification we in fact choose to implicitly assign profits that normally accrue to the payments received by capital. We realize that this specific assumption formally strengthens the results presented in this paper. However, the magnitude of the potential bias should not be significant as normally one can expect the correlation between capital income earned and profit income earned to be much higher than the correlation between labor income earned and profit income earned.

The magnitude of the labor force/employment is observable; accordingly we will choose to take values of L_t as given and will not model explicitly the labor supply decision. Moreover, in later stages we will assume that L_t includes only labor supplied by Polish citizens, which again formally will strengthen our results. Note, however, that changes in L_t that occurred in recent years were mostly driven by migration from Poland and not so much by arrivals of foreigners to Poland. Consequently, we believe that any potential bias introduced into our modeling by assuming that L_t reflects the decisions of Polish nationals is not significant.

Furthermore, it is true that in reality we observe movement of physical capital across borders. In particular, Poland has experienced significant capital inflows. Similarly, Polish nationals have invested in other countries. Nevertheless, the magnitude of capital inflows dwarfs the magnitude of capital outflows. Consequently, we can assume, without the risk of generating significant overestimates, that Polish investment abroad is equal to zero.

Recall that traditionally we choose to rely on the following accounting identity:

$$K_{t+1} = (1-\delta)K_t + I_t^F + w_t L_t + r_t K_t - C_t, \tag{12}$$

where δ denotes the rate of depreciation of physical capital, I_t^F denotes foreign capital inflows at time t, and C_t denotes the level of domestic consumption. Furthermore, one typically imposes particularly simple consumption decision rules such as a constant fraction of income being saved with the remainder consumed. Those consumption decision rules normally assume the form:

$$C_t = (1 - s)(w_t L_t + r_t K_t) = (1 - s)Y_t, \tag{13}$$

where s denotes the saving rate. In most applications the saving rate is assumed to be a parameter even though formally, the saving rate, s, is determined endogenously and constitutes an equilibrium variable. In our approach we allow for variations in s and normally identify the saving rate in a given time period with the empirically observed magnitude.

Allowing for the presence of foreign capital does not invalidate the accounting identity described with relationship (12). However, it calls for a more precise specification. Note that if foreign capital is present in a given economy, then of course we can write the following identity:

$$K = K^{PL} + K^F, \tag{14}$$

where K^{PL} denotes the magnitude of the capital stock owned by Polish nationals and K^F denotes the magnitude of capital stock present in Poland and owned by foreigners. Note that given this additional more precise specification we can rewrite accounting identity (12) as

$$K_t^{PL} + K_{t+1}^F = (1 - \delta)(K_t^{PL} + K_t^F) + I_t^F + w_t L_t + r_t(K_t^{PL} + K_t^F) - C_t. \tag{15}$$

Furthermore, we now have to acknowledge that the total earnings of Polish nationals take the form:

$$Y_t^{PL} = w_t L_t + r_t K_t^{PL}. \tag{16}$$

Observe that the above expression reflects our prior assumptions that labor is in its totality of domestic origin and that capital stock that exists in Poland is only partially owned by Polish nationals.

Recall that identities (12) and (15) are in fact equivalent. Nevertheless, we can still describe the actual evolution of capital stock in Poland with even higher precision. If we just focus on the capital stock that is owned by the domestic residents, then we can write the following identity:

$$K_{t+1}^{PL} = (1 - \delta)K_t^{PL} + w_t L_t + r_t K_t^{PL} - C_t. \tag{17}$$

Furthermore, we typically assume that $C_t = (1 - s_t)(w_t L_t + r_t K_t^{PL})$ where s_t denotes the saving rate of the domestic residents. Note that C_t denotes domestic consumption and as such faces a natural lower bound of zero, i.e., we always have $C_t \geqslant 0$.

The identity that describes the evolution of foreign capital stock present in Poland takes an even simpler form. In particular, we can write the following:

$$K_{t+1}^F = (1 - \delta)K_t^F + r_t K_t^F + I_t^F. \tag{18}$$

Recall that I_t^F denotes foreign capital inflows. Formally, I_t^F can be of either sign, i.e., there is no natural bound on I_t^F. Nevertheless, in Poland the value of I_t^F has been typically positive. We want to emphasize that our assumptions[3], thus, far in fact imply that I_t^F denotes the magnitude of new inflows of foreign capital as any retained profits are simply implicitly included in factor payment $r_t K_t^F$.

It is obvious that there is a discrepancy between income earned by Polish nationals and the total income generated in the Polish economy. In other words, we can be certain that there is a discrepancy between GDP and GNP in Poland. We are in a position now to assess the magnitude of this discrepancy. Recall that the level of GDP is given by equation (10) and the level of GNP is given by equation (16). Therefore, we can write the following:

$$GDP_t - GNP_t = (w_t L_t + r_t K_t) - (w_t L_t + r_t K_t^{PL}) = r_t(K_t - K_t^{PL}) = r_t K_t^F. \quad (19)$$

Not surprisingly the above identity confirms that the actual difference between the two measures of economic activity amounts to the earnings of foreign capital in Poland. How large are those earnings? We attempt to estimate the relevant magnitude next.

In order to obtain a more informative perspective let us express the discrepancy between the level of GDP and GNP in relative terms. Naturally, by definition we must have:

$$\rho = \frac{GDP_t - GNP_t}{GNP_t} = \frac{r_t K_t^F}{w_t L_t + r_t K_t^{PL}}, \quad (20)$$

which, given that $w_t = (1 - \alpha)A_t K_t^\alpha L_t^{-\alpha}$ and $r_t = \alpha A_t K_t^{\alpha - 1} L_t^{1 - \alpha}$, simplifies to

$$\rho = \frac{a K_t^F}{(1 - \alpha)K_t^F + K_t^{PL}} = \frac{\alpha}{(1 - \alpha) + \dfrac{K_t^{PL}}{K_t^F}}. \quad (21)$$

The discrepancy between the two measures of economic activity is essentially dictated by the ratio of the capital stock that is domestically owned, K^{PL}, to the capital stock that is foreign owned, K^F. To determine the actual evolution of the relevant ratio let us first denote the amount of foreign investment, I_t^F, in relative terms to the level of foreign income generated in Poland, $r_t K_t^F$, i.e., let us set $s_t^F r_t K_t^F = I_t^F$. Note that formally speaking there is no need to impose an upper bound on s_t^F as foreign investment can be financed from external sources. Under those assumptions the equations describing the evolution of the domestic capital and foreign capital – equations (17) and (18) – become

$$K_{t+1}^{PL} = (1 - \delta)K_t^{PL} + s_t(w_t L_t + r_t K_t^{PL}) \quad (22)$$

and

$$K_{t+1}^F = (1 - \delta)K_t^F + (1 + s_t^F)r_t K_t^F. \quad (23)$$

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[3] Note, that we also assume that consumption of foreigners residing in Poland is negligible.

Again, using the fact that $w_t = (1-\alpha)A_t K_t^\alpha L_t^{-\alpha}$ and $r_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}$ we can simplify the above conditions to

$$K_{t+1}^{PL} = (1-\delta)K_t^{PL} + s_t Y_t \left(1 - \alpha + \alpha \frac{K_t^{PL}}{K_t^F + K_t^{PL}}\right) \tag{24}$$

and

$$K_{t+1}^F = (1-\delta)K_t^F + (1+s_t^F)\alpha Y_t \frac{K_t^F}{K_t^F + K_t^{PL}}, \tag{25}$$

where of course $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$.

The above two conditions describe the evolution of the domestic capital stock and the foreign owned capital stock over time. We can use the above equations to trace the dynamics of the two capital stocks over time. Let us begin by defining the steady state outcomes. To simplify notation let us denote the steady state value of variable x_t with x_*. In this case we can rewrite the above two equations (note that by definition of the steady state $K_t^{PL} = K_{t+1}^{PL}$ and $K_t^F = K_{t+1}^F$) as

$$\delta K_*^{PL} = s_* Y_* \left(1 - \alpha + \alpha \frac{K_*^{PL}}{K_*^F + K_*^{PL}}\right) \tag{26}$$

and

$$\delta K_*^F = (1+s_*^F)\alpha Y_* \frac{K_*^{PL}}{K_*^F + K_*^{PL}}. \tag{27}$$

Dividing the above two equations we can establish that the following is true:

$$\frac{K_*^{PL}}{K_*^F} = \frac{s_*}{1+s_*^F}\left(\frac{1-\alpha}{\alpha} + \frac{1}{\alpha}\frac{K_*^{PL}}{K_*^F}\right), \tag{28}$$

which of course can be rearranged to yield

$$\frac{K_*^{PL}}{K_*^F} = \frac{\dfrac{s_*}{1+s_*^F}(1-\alpha)}{\alpha - \dfrac{s_*}{1+s_*^F}}. \tag{29}$$

Both saving rate s_* and s_*^F are in fact determined endogenously and reflect optimizing behavior of economic agents at home and foreign investors. We can imagine that foreign investors choose to allocate optimally their capital holdings and, in general, in response to higher rates of return earned in Poland choose to invest in Poland rather than elsewhere. Formally, we can expect that there is a relationship between the rate of return in Poland and abroad of the form $r_t = r_* + a$, where a denotes the risk premium and r_* is the rate of return abroad. Furthermore, we can expect that over time, given increased stability and increasing capital stock, the risk premium a will disappear and the that the two rates of return should be equalized across countries in the long run. Accordingly, we can write that in the steady state we can expect that $r_* = r_t$, which is equivalent to

$$r_* = \alpha A_* K_*^{\alpha-1} L_*^{1-\alpha} = \alpha \frac{Y_*}{K_*} = \alpha \frac{Y_*}{K_*^F + K_*^{PL}}. \tag{30}$$

We assume that the world interest rate is given and independent of any actions taken in Poland. Therefore, we can use the above equation to find the equilibrium value of the GDP in Poland in the long run, which is simply given by

$$Y_* = \frac{r_*}{\alpha}(K_*^{PL} + K_*^F). \tag{31}$$

Observe that we can now, given equation (31), express equation (27) as

$$\delta K_*^F = (1 + s_*^F)\alpha\frac{r_*}{\alpha}(K_*^{PL} + K_*^F)\frac{K_*^F}{K_*^F + K_*^{PL}}, \tag{32}$$

which of course simplifies to

$$(1 + s_*^F) = \frac{\delta}{r_*}. \tag{33}$$

Let us reiterate again that the foreign "saving" rate, s_*^F, reflects foreign private decision making. However, individual decisions of foreigners are made in line with the incentives foreigners face. In particular, the world interest, r_*, determines the composition of the foreign portfolio. Furthermore, when capital stock in Poland is composed of holdings by Polish national and holdings by foreigners, the actual foreign "saving" rate is pinned down by condition (33). Combining conditions (29) and (33), we can establish the following relationship:

$$\frac{K_*^{PL}}{K_*^F} = \frac{\frac{r_*s_*}{\delta}(1 - \alpha)}{\alpha - \frac{r_*s_*}{\delta}} = \frac{r_*s_*(1 - \alpha)}{\alpha\delta - r_*s_*}. \tag{34}$$

Now, combining equations (21) and (34) we can find the discrepancy between the Polish GDP and GNP in the long run as a function of the domestic saving rate s_*. The relevant ratio is given by

$$\rho_* = \frac{\alpha}{1 - \alpha + \frac{r_*s_*(1 - \alpha)}{\alpha\delta - r_*s_*}} = \frac{\alpha(\alpha\delta - r_*s_*)}{\alpha(1 - \alpha)\delta} = \frac{\alpha}{1 - \alpha}\left(1 - \frac{r_*s_*}{\alpha\delta}\right). \tag{35}$$

It is typically accepted that $\alpha = \frac{1}{3}$. Furthermore, rough estimates suggest that empirical values for the rate of depreciation and the world real net interest rate are given by $\delta = 0.07$ and $r_* - \delta = 0.035$, i.e., approximately we have $r_* = \frac{3}{2}$. Therefore, we can, based on those empirical estimates, express condition (35) as

$$\rho_* = \frac{1}{2}\left(1 - \frac{9}{2}s_*\right). \tag{36}$$

Similarly, we can now also rewrite condition (34) as

$$\frac{K_*^{PL}}{K_*^F} = \frac{r_*s_*(1 - \alpha)}{\alpha\delta - r_*s_*} = \frac{3s_*(1 - \alpha)}{2\alpha - 3s_*} = \frac{6s_*}{2 - 9s_*}. \tag{37}$$
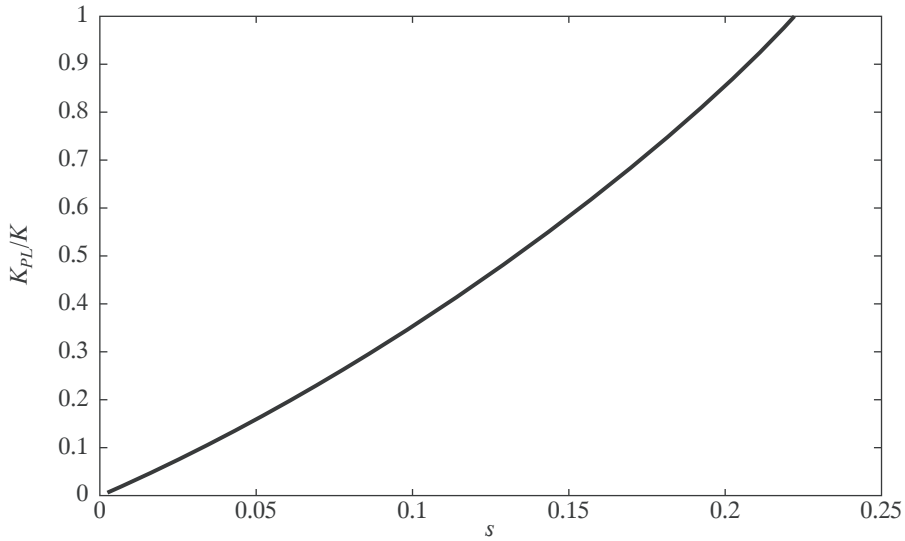
Naturally, we also have

$$\frac{K_*^{PL}}{K_*} = \frac{K_*^{PL}}{K_*^{PL} + K_*^F} = \frac{1}{1 + \dfrac{K_t^F}{K_t^{PL}}} = \frac{1-\alpha}{\alpha}\frac{r_* s_*}{\delta - r_* s_*} = \frac{6 s_*}{2 - 3 s_*}. \qquad (38)$$

We plot the discrepancy between the steady state values of the GDP and GNP and domestic to the overall capital ratio in Poland as functions of the domestic saving rate in Fig. (1a) and (1b).
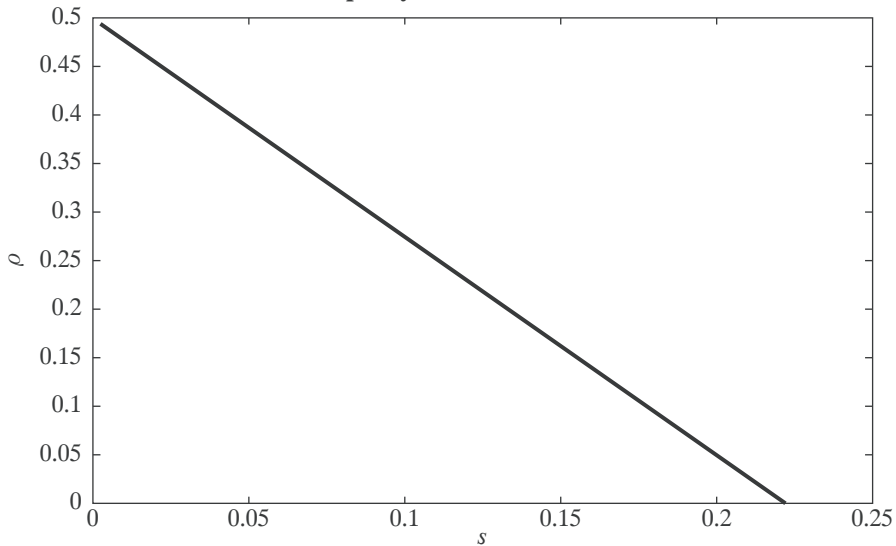
Figure 1a

The ratio of domestic to total capital

Figure 1b

The discrepancy between GDP and GNP

Before we proceed any further we would like to make several observations. First of all, given the values of the parameters we can state that there exists a steady state with foreign capital present in Poland only for a limited range of saving rates. In particular, if the domestic saving rate exceeds $\frac{\alpha\delta}{r_*} = 22\%$ then there is no long run equilibrium with foreign capital present in Poland. In such a case, with a significant domestic saving rate $s_* > \frac{\alpha\delta}{r_*}$, domestic capital accumulation occurs at a rapid pace. Consequently, the domestic capital stock becomes large and as a result the domestic rate of return becomes low discouraging foreign investment in Poland. Naturally, in such a case we can expect Polish capital to be moving abroad. However, we choose not to model this process explicitly in this paper.

Let us now consider the other extreme option. It could be the case that the domestic residents in Poland save relatively little. In such a case, when s_* is close to 0, in the long run domestic capital disappears completely. However, contrary to the traditional Solow model, output remains positive as there is foreign capital present that ensures that output is bigger than zero.

Our numerical simulations suggest that the degree of discrepancy between the GDP and GNP in Poland can be significant. As noted earlier, when the domestic saving rate is equal to $\frac{\alpha\delta}{r_*} = 22\%$ or more, then there is no incentive for foreigners to invest in Poland and in this case in fact GNP exceeds GDP. Again for obvious reasons we do not model this case explicitly. If the domestic saving rate is lower and equal to 20%, then the discrepancy between the GDP and GNP in the long run is equal to about 5%; when the saving rate is 15%, the discrepancy is equal to 16%; when the saving rate is 10%, then the discrepancy is equal to 27%; when the saving rate is 5%, then the discrepancy is equal to 38%; and finally, when the saving rate is equal to 0%, then the discrepancy is equal to 50%. In the final case GNP is in fact 50% lower than the GDP.

Naturally, the actual saving rate is a choice variable and is determined endogenously. At this stage we cannot predict the actual behavior of the domestic residents in Poland in the future. Hence, our analysis is conditional. Furthermore, it appears that currently the saving rate in Poland is equal to about 18%. Should this form of behavior continue, we can expect that asymptotically the discrepancy between the GDP and GNP in Poland will be about 9.5%, i.e., in the long run the GDP will be about 9.5% higher than the GNP. Naturally, our methodology can be used to assess the discrepancy for any other saving rate.

At this stage we would like to make several additional observations. First let us note that the discrepancy in the long implied by the current behavior of Polish residents can be substantial. As noted above, GDP will be about 9.5% larger than GNP or equivalently, GNP will be roughly 9.5% smaller than the GDP in the long run. To put things in perspective, let us note that assuming a modest annual rate of growth of 3.5%, the discrepancy between GDP and GNP amounts to about three years of growth, i.e., our calculations indicate that the actual standard of living of

Polish citizens lags the one implied by the officially published figures on GDP by about three years. Informally, we can state that the current statistics reflect the standard of living that will be attained by Polish nationals in 2021, and currently we enjoy the standard of living implied by the figures published in 2015.

We must add that our calculations are sensitive to the underlying assumptions. First of all, Poland is not in the steady state yet and using the steady state conditions can be only indicative and is probably highly premature. Moreover, the actual behavior can change and Polish nationals can choose to save more than 18%. Furthermore, the above calculations are based on the assumption that the average rate of growth in Poland is about 3.5%. We would like to point out that the implied lag would be equal to four years if we were to use a less optimistic value of 2.5%.

The above calculations pertain to GDP and GNP only. However, it is widely recognized that the primary objective of economic agents is slightly different than GDP or GNP Specifically, it is traditionally accepted that economic agents are likely to express interest in the level of consumption rather than in the level of overall economic performance. Taking the perspective of the domestic residents in Poland, we can express the amount consumed in Poland as

$$C_t^{PL} = (1 - s_t)Y_t^{PL}, \tag{39}$$

which of course in the steady state translates to $C_*^{PL} = (1 - s^*)Y_t^{PL}$.

By combining equation (26) and the steady state version of equation (7), we can establish that

$$\delta K_*^{PL} = s_* A_* K_*^{\alpha} L_*^{1-\alpha} \left(1 - \alpha + \alpha \frac{K_*^{PL}}{K_*^F + K_*^{PL}}\right), \tag{40}$$

which, given that translates to $K_* = K_*^{PL} + K_*^F$.

$$\delta K^{PL} = s_* A_* \left(\frac{K_*^F + K_*^{PL}}{K_*^{PL}}\right)^{\alpha} (K^{PL})^{\alpha} L_*^{1-\alpha} \left(1 - \alpha + \alpha \frac{K_*^{PL}}{K_*^F + K_*^{PL}}\right) \tag{41}$$

and further to

$$K_*^{PL} = \left(\frac{s_* A_*}{\delta}\right)^{\frac{1}{1-\alpha}} L_* \left(\frac{K_t^F}{K_t^{PL}} + 1\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \alpha + \alpha \frac{K_*^{PL}}{K_*^F + K_*^{PL}}\right)^{\frac{1}{1-\alpha}}. \tag{42}$$

The above equation determines the actual value of the domestic capital stock in the long run. Note that ratio $\dfrac{K_*^F}{K_*^{PL}}$ is known and given with condition (34).

Recall that we have the following relationship

$$Y_t^{PL} = w_t L_t + r_t K_t^{PL} = A_t K_t^{\alpha} L_t^{-\alpha} \left(1 - \alpha + \alpha \frac{1}{\dfrac{K_t^F}{K_t^{PL}} + 1}\right) \tag{43}$$

which translates to

$$Y_t^{PL} = A_t \left(\frac{K_t^F}{K_t^{PL}} + 1\right)^\alpha (K_*^{PL})^\alpha L_t^{1-\alpha} \left(1 - \alpha + \alpha \frac{1}{\frac{K_t^F}{K_t^{PL}} + 1}\right) \tag{44}$$

and further in the steady state – given equation (42) – to

$$Y_*^{PL} = \left(\frac{s_*}{\delta}\right)^{\frac{\alpha}{1-\alpha}} A_*^{\frac{1}{1-\alpha}} L_* \frac{\left(1 + (1-\alpha)\frac{K_*^F}{K_*^{PL}} + 1\right)^{\frac{1}{1-\alpha}}}{1 + \frac{K_*^F}{K_*^{PL}}}. \tag{45}$$

Again, let us recall that condition (34) holds. Thus, we can rewrite the above equation as

$$Y_*^{PL} = \left(\frac{s_*}{\delta}\right)^{\frac{\alpha}{1-\alpha}} A_*^{\frac{1}{1-\alpha}} L_* \frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\delta^{\frac{\alpha}{1-\alpha}}}{(\delta - r_* s_*)(r_* s_*)^{\frac{\alpha}{1-\alpha}}} = \frac{\delta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A_*^{\frac{1}{1-\alpha}} L_*}{(\delta - r_* s_*)(r_*)^{\frac{\alpha}{1-\alpha}}}. \tag{46}$$

Therefore, the level of consumption in the steady state in Poland is given by

$$C_*^{PL} = (1 - s_*)Y_t^{PL} = (1 - s_*)\frac{\delta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A_*^{\frac{1}{1-\alpha}} L_*}{(\delta - r_* s_*)(r_*)^{\frac{\alpha}{1-\alpha}}}. \tag{47}$$

Note that we allow for the presence of foreign capital, i.e., the actual output is higher than Y_*^{PL} and equal to $Y_* = K_*^{PL} + r_* K_*^F$. Naturally, Polish nationals are not entitled to consume out of $r_* K_*^F$. Nevertheless, we can hypothetically ask what the consumption would be if Polish nationals were allowed to. In such a case the level of consumption would be

$$C_*^H = (1 - s_*)Y_*. \tag{48}$$

Naturally, this hypothetical consumption level would be higher and the discrepancy between the hypothetical level and the actual level would be correspond to the level of discrepancy between the GDP and GNP.

Finally, note that the saving rate influences the level of consumption in the steady state. In particular, consumption in the steady state increases with the saving rate as figure (2) indicates.

The steady state value of consumption increases with the saving rate. Therefore, Polish nationals could increase their level of consumption in the steady state by increasing the saving rate. In particular, if the current saving rate of 18% were to be replaced with saving rate of 22% then consumption in the steady state would increase by 4%. Observe, however, that the saving rate of 22% in fact implies that

Figure 2

The steady state value of consumption as a function of the saving rate
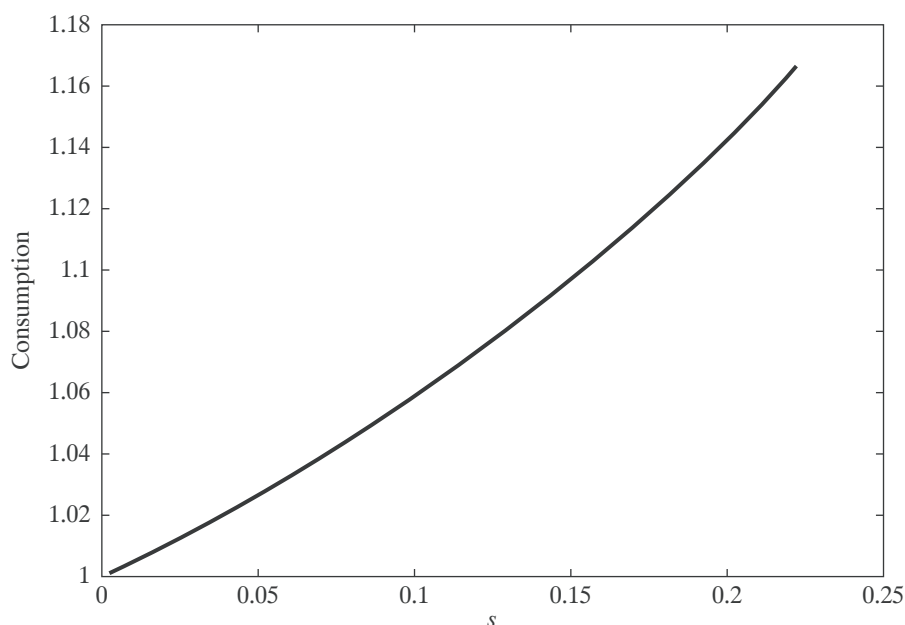
there is no foreign capital in Poland in the long run. Consequently, we can state that the presence of foreign capital in Poland in the long run is detrimental to consumption and cannot lead to welfare improvement unless there are technological spillovers. We will explore this opportunity in subsequent research.

Conclusions

In this paper we show that in Poland there will be a discrepancy between the GDP and GNP in the long run. Specifically, we show that the inflows of foreign capital to Poland are likely to stabilize assuming that the saving rate in Poland will not exceed 22%. Furthermore, we illustrate, given the presence of foreign capital in Poland in the long, that the discrepancy between the GDP and GNP in Poland will be equal to about 9.5% under the assumption that Polish nationals will not change their saving behavior. Furthermore, we argue that the long run level of consumption will not be maximized unless the saving rate in Poland is adjusted sufficiently to discourage foreign inflows of capital. Our estimates suggest that Polish nationals could increase their consumption in the long run by about 4% by increasing the saving rate to 22%.

Our principal findings reveal that the standard of living as measured by the traditional measure of the level of economic activity, GDP, can be in fact over-

estimated. We suggest that focusing on other measures such as GNP can be more relevant in policy design and policy evaluation. Furthermore, we note that the Polish experience in the last fifteen years has not be as stellar as normally believed given the experiences of countries neighboring Poland. This simple observation becomes particularity relevant in the context of significant inflows of foreign capital to Poland and to a much lesser extent to successful neighbors of Poland such as Russia.

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EQUILIBRIUM DISCREPANCY BETWEEN GDP AND GNP IN POLAND

S t r e s z c z e n i e

W artykule tym autor próbuje oszacować długookresową różnicę między wartością PKB i PNB w Polsce za pomocą teoretycznego modelu opartego na funkcji produkcji Cobba-Douglasa. Uzyskane wyniki sugerują, że w długim okresie PKB w Polsce może być o 9,5% wyższy od PNB, co oznaczałoby, że faktyczny poziom dochodów Polaków będzie opóźniony o około 3 lata względem PKB. Ponadto autor sugeruje, że rzeczywisty poziom konsumpcji w Polsce będzie maksymalny w długim okresie w przypadku całkowitej nieobecności kapitału zagranicznego i że konsumpcja w Polsce byłaby wówczas o około 4% wyższa dzięki obniżce stopy procentowej spowodowanej przez wyższą stopę oszczędności.

Słowa kluczowe: PKB, PNB, funkcja produkcji, równowaga długookresowa, inwestycje zagraniczne, Polska

JEL: F21, F43, F62

EQUILIBRIUM DISCREPANCY BETWEEN GDP AND GNP IN POLAND

S u m m a r y

In this paper a theoretical approach, based on Cobb-Douglas production function, is used to assess the difference between the GDP and GNP values in Poland. The results suggest that in the long run GDP in Poland will be about 9.5% higher than GNP implying that the actual income levels enjoyed by Polish nationals will be lagging those implied by GDP by about three years. Furthermore, the author suggests that the actual consumption levels in Poland will be maximized in the long run when foreign capital is fully absent, and that consumption in Poland could be increased by about 4% when foreign ownership of capital is fully absent, thanks to the reduction in the domestic interest rate driven by higher saving rates.

Key words: GDP, GNP, production function, long run equilibrium, foreign investment, Poland

JEL: F21, F43, F62

РАЗЛИЧИЯ МЕЖДУ ВВП И ВНП В ПОЛЬШЕ

Р е з ю м е

В статье автор пытается рассчитать долгосрочную разницу между значениями ВВП и ВНП в Польше с помощью теоретической модели, использующей функцию продукции Кобба-Дугласа. Полученные результаты показывают, что в длительный период значение ВВП может быть выше, чем ВНП на 9,5%, что означало бы, что фактический уровень доходов граждан примерно на 3 года отстает от доходов, вытекающих их ВВП. Кроме того, автор полагает, что действительный уровень потребления в Польше был бы максимальным в длительный период в случае полного отсутствия иностранного капитала. Потребление в Польше было бы тогда примерно на 4% выше благодаря понижению процентной ставки, вызванной более высокой нормой сбережений.

Ключевые слова: ВВП, ВНП, функция продукции, долгосрочное равновесие, иностранные инвестиции, Польша

JEL: F21, F43, F62